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# PHYSICAL LIMITATION ON MAXIMAL GRADIENT IN LINEAR ACCELERATOR

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## ABSTRACT

We investigate here the limits on maximal gradient achievable in a linear accelerator. In particular the possible influence of Compton scattering of accelerated electrons or positrons on the photons in accelerating structure excited on fundamental frequency. This interaction expressed in terms of effective length at which the probability of scattering goes to unity. This effect may bring limiting gradient to about few GeV per meter for accelerating wavelength $\lambda \cong 1\mu m$ . The same process might be responsible for background effects in planning linear colliders at energy $\leq 250\ GeV$ .

## INTRODUCTION

Investigation of limits in maximal gradient achievable is important in connection with investigation of laser driven linac possibility [1]. One evident limitation what appears here–is the breakdown limit for the material of the accelerating structure. Quantum mechanical considerations done gives a 10 *GeV/m* as a limiting value in this case. Beyond this we suggest to consider also a possible limitations arising from interaction of the accelerating electron with the photons of main frequency in the cavity. It is now a common place to talk that the linear accelerator has very small power losses due to the fact that the acceleration has the same direction as the speed of particle. This fact was established many years ago, when the gradients were small and the limitation associated with this effect is negligible. It is true, that the power losses are much less here, compared with transverse acceleration, but nobody made estimation what is the real limit in case of linear accelerator. It is time to review this in connection with search of the maximal gradients achievable. For this considerations we treat the accelerating field as a coherent state for the photons in a cavity. Electrons are scattered on the photons of the cavity during acceleration. This interaction between electrons and photons is inevitable, as the same photons are responsible for acceleration and scattering at the same time. So the consideration done here deals with regular photons at fundamental frequency of the cavity.

In terms of quantum mechanics all these photons are in the same state and the number of photons is simply equal to the filling number for this state. One fundamental difference

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between free photon and the photon in the cavity is the following. Free photon could be represented as a plane wave[1]. Meanwhile the photon in the cavity is a superposition in simplest case of two plane waves. The last superposition is a standing wave arranged to satisfy the boundary condition. As the gradients in a laser driven structure is extremely high, the number of the photons excited by external source is big also. So effective length of interaction $l_\gamma \cong 1/n_\gamma\sigma_\gamma$, where $n_\gamma$–is the volume photon density and $\sigma_\gamma$–is effective cross section of electron-photon interaction, might became comparable with characteristic size of linear accelerator or with the effective length $l_s \cong c\tau_s$, where $\tau_s$–is the characteristic damping time in a storage ring.
The interaction of the accelerated particles with the photons in accelerating structure of next generation linear collider also creates problems, what look fundamental.
This effect might be a source of narrow band X-ray radiation from the cavity in a storage ring also. At least here this effect might be useful (for X-ray spectroscopy for example).

## INTERACTION WITH THERMAL PHOTONS

The Compton scattering of electrons on photons became interesting on macroscopic level since the scattering of electrons by thermal photons of Sunlight and starlight was considered in [2]. It was suggested here, that this mechanism might be responsible, in part, for the energy leveling of cosmic electrons[2]. These ideas were applied later in [3] to a storage ring area of interest. It was shown here, that high-energy electrons and positrons might be tolerated by the thermal photons, which are present in a vacuum chamber of the ring.
Let us begin with the comment on electron interaction with thermal photons. Thermal photons are staying in equilibrium with the walls by absorbing/emitting process. They are treated as statistically independent i.e. having random phase distribution and polarization at every region of their spectra. The photon density well described by Plank formula [4]. Usually the fundamental frequency is much lower at room temperatures, than the one corresponding to the maximal energy in thermal photon spectra. With other words $\lambda << c/\omega_0$, where $\omega_0$ is the fundamental frequency of the cavity, $c$ –is a speed of light[3]. So, modification of the spectrum of radiation, due to its cut at the fundamental frequency of the cavity, is not tolerating the spectrum.
The peculiarity may appear when fundamental frequency of the cavity becomes *higher* than corresponding frequency for thermal photons. This is the case when the structure has a fundamental frequency close to the corresponding frequency in thermal spectrum. This may happen, for example, in microstructures, designed for infrared region power source ($\approx 1\mu m$ laser wavelength), especially if they are cooled down to cryogenic temperatures. Such microstructures are under consideration for a laser driven accelerator, see [1] for example. This size-effect suppresses the thermal radiation of the walls and, sequentially, the number of thermal photons in lower part of the spectra. At high edge of the spectra there is usual exponential suppression. The photons in the region of fundamental frequency are not *real* under this circumstance. So the scattering of accelerated beam on thermal photons becomes

[1] Strictly speaking the laser waist is spherical (or cylindrical) wave. Laser wavelength in the waist is larger, than far from it.
[2] The sun delivers about $\approx 2\cdot 10^7$ photons/cm$^3$ with $\cong 1.35\ eV$ what brings $\cong 2.7\cdot 10^7 eV/cm^3$ at the earth.
[3] This means that number of states occupied is large.

negligible here.
In this publication the similar ideas applied however to electron scattering on the photons of the *RF cavity* or *accelerating structure*. The last one excited by external power source on fundamental frequency of the cavity. The consideration made here initiated in connection with a laser driven linac. In such linac the microstructures excited to very high level, typically close to the break down limit of the material of the structure. So the number of the photons is high. The same high is the volume photon density. Consideration done shows that break down limit for the material is close to the limit, defined by the scattering of the electrons on the photons for $\approx 1\mu m$ laser wavelength. These limitations are responsible for the maximal gradient allowable in a laser driven accelerating structure.
The quantum effects could be important also for cm wavelength accelerating structure also.

## COHERENT PHOTONS IN A CAVITY

In contrast to thermal photons, the photons in a cavity excited by external source on *fundamental* frequency are coherent under this condition of excitation. All they have the same phase and polarization.
Let $q$ be the energy stored in a cell/cavity on the lowest (fundamental) frequency $\omega_0$. Sequentially, the number of the quants here is $N_\gamma \cong q/\hbar\omega_0, \hbar$ –is the Plank constant. If the quality factor of the cell is $Q_{RF}$, then the external power keeping this number of photons is $P_{ext} \cong q\omega_0 / 2\pi Q_{RF}$. The electric field strength is $E \sim \sqrt{N_\gamma / V}$ where $V$ –is the volume of the cavity, and could be defined from equation $\varepsilon_0 \int_V E^2 dV \cong q$, $\varepsilon_0 \cong \frac{1}{36\pi} 10^{-9} F/m$. The last means that the energy of quanta is normalized to the volume of the cavity, having dimensions $V \cong L_x \times L_y \times L_s$. Transverse size of the cavity defined usually by dispersion equation $k_x^2 + k_y^2 + k_s^2 = \omega_0^2 / c^2$, where $k_{x,y,s} \cong 2\pi / L_{x,y,s}$–are the wave vectors. We can estimate it roughly[4] $V \cong \lambda_W{}^3 / 8$, where $\lambda_W$ –is the effective wavelength in the cavity defined by dispersion equation. In free space also the wavelength is higher in a waist. So

$$q = \int_v \frac{(\varepsilon_0 E^2 + \mu_0 H^2)}{2} dV \cong \frac{\varepsilon_0 E^2 \lambda_W^3}{16}$$

where $\mu_0 = 4\pi \cdot 10^{-7} H/m$ is magnetic permeability of vacuum. We suggested also that the height of the cavity in direction of the beam propagation be exactly $\lambda_W / 2$. Electric field strength goes to

$$E \cong 4\sqrt{\frac{q}{\varepsilon_0 \lambda_W^3}}$$

The number of the photons could be found as

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[4] For cylindrical cavity $V \cong \pi a^2 h$, where $a \cong 2.405\lambda_0 / 2\pi$ is the radius of the cavity, $h$ –is the height of the cavity, $h \cong \lambda_0 / 3$, so $V \cong \lambda_0^3 / 6.54$, but $q \cong 2\pi\varepsilon_0 E_0^2 h \int_0^a J_0^2(k_r r) r dr$.

$$N_\gamma \cong \frac{1}{16}\frac{\varepsilon_0 E^2 \lambda_W^3}{\hbar\omega_0} \cong \frac{1}{16\pi}\frac{\varepsilon_0 E^2 \lambda_W^4}{\hbar c}.$$

The photon density goes to

$$n_\gamma \cong \frac{8\cdot N_\gamma}{\lambda_W^3} \cong \frac{8}{16}\frac{\varepsilon_0 E^2}{\hbar\omega_0} \cong \frac{1}{2\pi}\frac{\varepsilon_0 E^2 \lambda_W}{\hbar c}.$$

One can see that the photon density is dropping linearly with the wavelength if $E$ is fixed. Let us now calculate the power required for excitation the accelerating structure having the total length $L$ and the wavelength $\lambda_0 \cong 1\mu m$. On the distance $L$ along the linac, the number of the cells goes[5] to $N_{cell} \cong L/\lambda_0$, so at the distance, say $L$ =3 *cm*, the last number goes to $N_{cell} \cong 3\cdot 10^4$ cells. The total energy stored in these cells goes respectively to $Q \cong qL/\lambda_0 \cong 1.6\cdot 10^{-9} J$. If the duty of feeding radiation lasts for a time $\tau$, then the last amount must be multiplied by the factor $c\tau/\lambda_0$. In TLF method [1] the last factor goes to ~100. So total energy required from the laser flush goes to

$$Q \cong \frac{qL}{\lambda_0}\times\frac{c\tau}{\lambda_0} \cong q\cdot\left(\frac{L}{\lambda_0}\right)^2 \times \chi,$$

where factor $\chi \approx w_\perp/\lambda_0$ reflects the ratio of the width of illuminating transverse area to the wavelength. This parameter could be expected of the order $\chi \approx 5$. The total energy goes to $Q \cong 5.5\cdot 10^{-13}\left(3\cdot 10^4\right)^2 \times 5 \cong 247\cdot 10^{-5} J \cong 2.5 mJ$. So this 2.5*mJ* flash could feed ~3*cm* structure, delivering 30*MeV*.

The damage level to the structure by a laser radiation is strongly correlated with the duration of illumination. In TLF method this energy about $Q \cong 2.5mJ$ effectively distributed on the area $S \cong w_\perp \cdot L$ in the time of duration $\tau$. So it gives density of energy falling on this area as high as $Q/S \cong Q/w_\perp L \approx 2.5\cdot 10^{-3}/5\cdot 10^{-4}/3 \cong 1.6 J/cm^2$ for $\lambda_0 \cong 1\mu m$. The illumination at local point lasts for a time $t \cong l_t/c \cong l_f/c \cong 10^{-13} s$ in TLF method [1]. Using the quality factor $Q_{RF} \cong 5$ the laser energy could be decreased in the same proportion.

Classical 4-potential corresponding to the field $E$, could be represented as $A = \{A_\nu\} \equiv \{-\vec{n}E/i\omega_0, 0\}$, $\vec{n}$–is unit vector. The dimensionless parameter characterized the interaction of the electron with electromagnetic wave is ([4], p. 449)

$$\xi = \frac{e\sqrt{-A^\nu A_\nu}}{mc} \cong \frac{eE\lambda}{2\pi mc^2}.$$

Basically this could be treated as a deflection parameter, similar to the broadly used deflection parameter for magnetic field[6], $K = \dfrac{eH\lambda}{2\pi mc^2}$. So $\beta_\perp \cong \xi\gamma$ is a measure of transverse deflection of scattered electron. For specific polarization of electromagnetic wave in a cavity along the electron trajectory, the energy gained by the particle into the cavity is about $eU \cong eE\lambda$. So

[5] Standing wave structure.
[6] In Gaussian units these parameters have the same value, obviously.

condition for $\xi$ means that the energy gained by the particle in the cavity is less compared with the rest energy of electron. There are two extreme cases emerge here. First one is corresponding to $\xi >> 1$ and is appropriate to the nonlinear interaction [4]. The spectrum of radiation could be found from the formulas of synchrotron radiation in strong quantum regime. Opposite case $\xi << 1$ is appropriate to a single photon, or Compton interaction. In our case substitute here the same parameters for estimation, $E \cong 1GeV/m$, $\lambda_W \cong 10^{-6}m \equiv 1\mu m$, one can obtain

$$\xi \cong \frac{E\lambda}{2\pi(mc^2/e)} \cong \frac{10^9 \cdot 10^{-6}}{2\pi \cdot 510 \cdot 10^3} \cong 3 \cdot 10^{-4} << 1 .$$

So the process under our interest could be described as a *Compton scattering*.

For this example the energy gain in the gap of the cavity is $eU \cong 10^3 eV << 510 keV$. So the number of absorbed photons goes to $eU/\hbar\omega_0 \cong 10^3$ in this example. This means that the process could be described well from the point of classical electrodynamics. This is a basic principle of correspondence in quantum electrodynamics [4].

Let us consider few other examples. Let first as above $E \cong 1GeV/m$, $\lambda_W \cong 10^{-6}m \equiv 1\mu m$, then the energy of the photon goes to

$$\hbar\omega_0 = 2\pi\hbar c/\lambda_W \cong 1.98 \cdot 10^{-19} J \cong 1.24 eV .$$

The energy stored in a cell excited to this level of electric field strength goes to

$$q = \int_v \frac{(\varepsilon_0 E^2 + \mu_0 H^2)}{2} dV \cong \frac{\varepsilon_0 E^2 \lambda_W^3}{16} \cong 5.5 \cdot 10^{-13} J$$

The number of quants in a cell becomes $N_\gamma \cong 2.8 \cdot 10^6$. The photon density in this example is

$$n_\gamma \cong \frac{8 \cdot N_\gamma}{\lambda_W^3} \cong \frac{8}{16}\frac{\varepsilon_0 E^2}{\hbar\omega_0} \cong 2.24 \cdot 10^{25} 1/m^3 \equiv 2.24 \cdot 10^{19} 1/cm^3$$

Let us consider now NLC/SLAC as example. Let $E \cong 30MeV/m \equiv 0.03GeV/m$, $\lambda_W \cong 10^{-1}m \equiv 10cm$, then, first

$$\hbar\omega_0 = 2\pi\hbar c/\lambda_W \cong 2\pi 1.054 \cdot 10^{-34} 3 \cdot 10^{10}/10 = 1.98 \cdot 10^{-24} J \cong 1.24 \cdot 10^{-5} eV .$$

The energy stored in a cell goes to

$$q = \int_v \frac{(\varepsilon_0 E^2 + \mu_0 H^2)}{2} dV \cong \frac{\varepsilon_0 E^2 \lambda_W^3}{16} \cong \frac{10^{-9} \cdot 10^{14} \cdot 0.001}{4\pi 16} = 0.5J$$

The number of quants becomes $N_\gamma \cong q/\hbar\omega_0 \cong 0.5/1.24 \cdot 10^{-24} \cong 4 \cdot 10^{23}$. The photon density in this example is

$$n_\gamma \cong \frac{8 \cdot N_\gamma}{\lambda_W^3} \cong \frac{32 \cdot 10^{23}}{0.001} \cong 3.2 \cdot 10^{27} 1/m^3 \equiv 2.24 \cdot 10^{21} 1/cm^3 .$$

One can see that the photon density is extremely high. Namely this high density might be responsible for intense electron-photon interaction, see lower.

## COMPTON (SINGLE PHOTON) SCATTERING

Let $\hbar\omega_0$, $\hbar\vec{k}_0$ are the energy and momentum of incoming photon, $\hbar\omega'$, $\hbar\vec{k}'$ –the energy and momentum of the backward scattered gamma-quanta. Then these parameters linked by the formula[7] [4]

$$\omega_0(1-\frac{v}{c}Cos\vartheta)-\omega'(1-\frac{v}{c}Cos\vartheta')=\frac{\hbar\omega_0\omega'}{mc^2\gamma}(1-\frac{v}{c}Cos\hat{\vartheta}) \ ,$$

where $v=|\vec{v}|$ and $mc^2\gamma$ are the initial velocity and the energy of incoming electron, $\vartheta$ and $\vartheta'$ are the angles between $\vec{v}$ and wave vectors $\vec{k}_0$ and $\vec{k}'$ respectively, $\hat{\vartheta}$ is the angle between $\vec{k}_0$ and $\vec{k}'$, see Fig. 1.

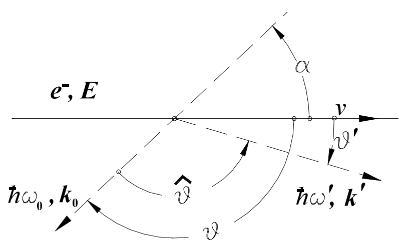


**FIGURE 1:** Cinematic of the Compton scattering.

Introducing the angle $\alpha$ between the line of incoming photon and the vector $\vec{v}$ as it is shown on the Fig.1, and supposing that $\vartheta' \approx 1/\gamma$, one can obtain the energy of the backward scattered photon

$$\hbar\omega'=\frac{mc^2\gamma\cdot X\cdot Cos^2(\alpha/2)}{1+\gamma^2\vartheta'^2+XCos^2(\alpha/2)}=\frac{\hbar\omega'_{\max}}{1+\dfrac{\gamma^2\vartheta'^2}{1+X\cdot Cos^2(\alpha/2)}} \ ,$$

where

$$X=\frac{4\gamma^2\hbar\omega_0}{mc^2\gamma}=\frac{4\gamma\hbar\omega_0}{mc^2},\ \hbar\omega'_{\max}=\frac{4\gamma^2\hbar\omega_0\cdot Cos^2(\alpha/2)}{1+X\cdot Cos^2(\alpha/2)}=mc^2\gamma\frac{x}{1+x},$$

$x=X\cdot Cos^2(\alpha/2)$. One can see, that dimensionless parameter $X$ is proportional to the ratio of the photon energy in the electron rest frame to the electron's rest energy. In the case of RF cavity or accelerating structure under interest, $\alpha=\pi/2$, and $x=X/2$.

Let us make estimations for two examples considered in previous Chapter. For the first example let the $\hbar\omega_0 \cong 1.17\ eV$ (wavelength $\lambda_0 \cong$ 1.06 $\mu m$, Neodymium glass laser, this is a little bit more precise wavelength), $mc^2\gamma$=250 $GeV$ ($\gamma \cong 5\cdot 10^5$), so $X\cong 4.5$, $x\cong 2.25$ and maximal possible energy of the photon is $\hbar\omega'_{\max}\cong mc^2\gamma\cdot 4/10\cong 170GeV$ at this energy of electron.

For the second example let us consider $mc^2\gamma$=50 $GeV$ ($\gamma\cong 10^5$), SLAC, $\hbar\omega_0\cong 1.24\cdot 10^{-5}eV$, $X\cong\frac{4\gamma\hbar\omega_0}{mc^2}\cong 9.7\cdot 10^{-6}$, $x=X/2\cong 4.8\cdot 10^{-6}$ and $\hbar\omega'_{\max}\to 5\cdot 10^7 keV\cdot 4.8\cdot 10^{-6}\cong 243keV$.

The formula for the photon energy can be represented also as the following

[7] For $\vec{v}=0$ well known Compton formula.

$$\hbar\omega' = \frac{\hbar\omega'_{\max}}{1+\dfrac{\gamma^2\vartheta'^2}{1+X\cdot Cos^2(\alpha/2)}} = \frac{\hbar\omega'_{\max}}{1+\dfrac{\vartheta'^2}{\vartheta_0^2}},$$

where the effective angle $\vartheta_0 = \sqrt{\dfrac{1+x}{\gamma^2}}$ was introduced. When the scattering angle $\vartheta' \cong \vartheta_0$, the energy of the quanta is a half of the maximal possible. In first example above it happens when $\vartheta_0 = \frac{\sqrt{3}}{5}10^{-5} rad$. For SLAC this is five times higher.

If the invariant mass of the photon and the gamma quanta $E_{inv} = \sqrt{4\hbar\omega_0\hbar\omega'}$ is bigger, than the electron-positron pair rest mass $E_{inv} \ge 2mc^2$, then the electron-positron creation can occur. This value is $X \ge 2(1+\sqrt{2}) = 4.8$.

The energy spectrum of the scattered unpolarized photons is defined by the formula [4]

$$\frac{1}{\sigma_C}\frac{d\sigma_C}{dy}\bigg|_u = \frac{2\sigma_0}{x\sigma_C}\left[1-y+\frac{1}{1-y}-\frac{4y}{x(1-y)}+\frac{4y^2}{x^2(1-y)^2}\right],$$

where $\sigma_0 = \pi r_0^2 = \pi(e^2/mc^2)^2 \cong 2.5\cdot 10^{-25}\ cm^2$, $y = \hbar\omega'/mc^2\gamma$. Total Compton cross section $\sigma_C$ could be obtained by integration of previous formula as

$$\sigma_C = \frac{2\sigma_0}{x}\left[\left(1-\frac{4}{x}-\frac{8}{x^2}\right)\log(1+x)+\frac{1}{2}+\frac{8}{x}-\frac{1}{2(1+x)^2}\right].$$

The last formula could be expanded in two extreme cases as

$$\sigma_C = \frac{8\sigma_0}{3}(1-x), \quad x<<1,$$

$$\sigma_C = \frac{2\sigma_0}{x}(\log x+\frac{1}{2}), \quad x>>1.$$

For our two examples $\sigma_C = 2.98\cdot 10^{-25} cm^2$ for $mc^2\gamma$=250 *GeV*, and $\sigma_C = 6.65\cdot 10^{-25} cm^2$ for $mc^2\gamma$=50 *GeV*.

For future reference let us calculate the cross section for $mc^2\gamma$=2 *TeV*, $\gamma \cong 4\cdot 10^6$. In this case $X \cong \dfrac{4\gamma\hbar\omega_0}{mc^2} \cong 36.7$, $x \cong 18.3$, $\sigma_C = \dfrac{2\sigma_0}{36.7}(\log 36.7+\dfrac{1}{2}) \cong 0.22\sigma_0 \cong 5.58\cdot 10^{-26} cm^2$. In this case also $\hbar\omega'_{\max} \cong mc^2\gamma\dfrac{x}{1+x} \cong 0.97 mc^2\gamma \cong 1.95 TeV$.

In the case of our interest the polarization of the photon $\xi_3$ = -1 is going along the accelerating field. So the polarization is always stay in the scattering plane. The differential cross section for the unpolarized electrons[8] could be described by the formula [4]

[8] I.e. electrons without interest to their polarization.

$$d\sigma(\xi_3) \cong \frac{1}{2} r_0^2 \cdot \left(\frac{\omega'}{\omega_0}\right)^2 \cdot \left(\frac{\omega_0}{\omega'} + \frac{\omega'}{\omega_0} - (1-\xi_3)\cdot Sin\hat{\vartheta}\right) do' .$$

For polarization perpendicular to the scattering plane $\xi_3 = 1$. So the dependence on polarization is weak, as we are interesting in the case when $\omega'/\omega >> 1$.

## INTERACTION

Let us first review the formula for cross section from the classical point of view. The formula for the power losses could be represented as the following

$$I \cong \frac{dmc^2\gamma}{dt} = \frac{2e^2}{3c^3}\gamma^6 \cdot (w^2 - (\vec{\beta}\times\vec{w})^2) ,$$

where acceleration $w$ in our case is $w \cong eE/m\gamma^3$ as the force is a longitudinal one. The intensity of radiation is zero in a straightforward direction. Substitute the last formula in previous expression one can obtain

$$I \cong \frac{dmc^2\gamma}{dt} = \frac{2e^2}{3c^3}\gamma^6 \cdot \frac{e^2E^2}{m^2\gamma^6} = \frac{2r_0^2}{3}\cdot E^2 c .$$

In terms of scattering language if we divide this expression by the Pointing vector of the photons we obtain a cross section of the process, $\sigma \cong \frac{I}{S}$. The flux is $S \cong \frac{c}{4\pi}E^2\rho$, where $\rho \cong 1 \div 2$ is a factor reflecting the so-called conductance of the cavity, what is basically a fraction of the inverse free space impedance. We will put this $\rho \cong 1$ for simplicity. As a result we come to the same cross section

$$\frac{I}{S} \cong \frac{2r_0^2}{3}\cdot E^2 c \frac{4\pi}{cE^2} \cong \frac{8\pi}{3} r_0^2 .$$

So in treatment of the process in terms of a cross section there is no difference between the polarization of the photon. The radiation is absent in straightforward direction. This is evident from the rest frame of electron. Acceleration in this frame is longitudinal and electric field polarized also along the direction of motion. Dipole radiation is absent in polar direction[9]. The angular distribution of radiation can be represented as the following

$$\frac{dI}{do} = \frac{e^2}{4\pi c^3}\cdot\left(\frac{eE}{m\gamma^3}\right)^2 \frac{Sin^2\vartheta'}{(1-\beta Cos\vartheta')^6} .$$

Differential cross section could be obtained from this expression by division it by $S \cong \frac{c}{4\pi}E^2\rho$, $\frac{d\sigma}{do} \cong \frac{r_0^2}{\gamma^6}\cdot\frac{Sin^2\vartheta'}{(1-\beta Cos\vartheta')^6} \cong \frac{2^6 r_0^2\cdot\gamma^6\vartheta'^2}{(1+\gamma^2\vartheta'^2)^6}$.

In the rest frame of electron the accelerating gap looks squeezed by a factor $\gamma$, so the radiation in this frame lasts for a time $\sim \lambda / c\gamma$ what defines the characteristic frequency $c\gamma/\lambda \cong \omega_0\gamma$. Transformed into Lab frame this gives the characteristic frequency $\sim \omega_0\gamma^2$. One

[9] Condition for dipole radiation (beam size) $<< \lambda$ is well satisfied here.

can obtain this in a different way by mention that the time duration in Lab frame is $\sim \lambda / c$. This time squeezed further by Doppler effect. So one can conclude that the quantum effects becomes strong, when $\hbar\omega_0\gamma^2 \leq mc^2\gamma$, what means the energy of the quanta becomes close to the rest energy of electron. This requirement is the same as expressed through $x$ –parameter.
Radiation has a maximum at the angle $\vartheta_0 = \sqrt{\frac{2}{5}(1-\beta)} \cong 1/\sqrt{5}/\gamma$ and has angular spread $<\vartheta> = 1/\gamma$. The instant view of radiation looks like a cone with angle at the top $\vartheta_0$ and each wall having the angular divergence $<\vartheta> = 1/\gamma$. So basically the quantum energy need to be taken as a half of maximal.
In this connection let us made a comparison of the losses with the energy gain as the following. As the radiation is a dipole one, the particle does not losses the momenta in its rest frame, so

$$-\frac{d\vec{p}}{dt} = \frac{\vec{v}}{c^2} I = \frac{\vec{v}}{c^2} \cdot \frac{2}{3} r_0^2 E^2 c \cong eE \ .$$

The last formula represents the balance between the losses and gain in case of linear acceleration. The last formula gives

$$E \cong \frac{3}{2}\frac{e}{r_0^2} \text{ (CGS units).}$$

This one can expect from the formulas of classical electrodynamics. Quantum effects appears at distances at least $1/\alpha \cong 137$ times larger, than in classical case. Let us estimate the field however. Substitute in last formula the values, $E \cong 2.75 \cdot 10^{20} V/m$. Quantum mechanical limit goes to ~$2\alpha$ times lower as the Schwinger field is $E^{\infty} \cong (2mc^2/e)/(r_0/\alpha)$.
In [5] the scattering of electron in the field on standing electromagnetic field was considered. The authors found also the absence of radiation in forward direction. The explanation given later on treats this as a Bragg conditions in scattering of electron on the grating what the standing wave represents. Further this effect was used for the profile measurements of relativistic beam [6].
We are continuing the description in terms of the photon-electron interaction. As the photon density is known, it is possible to calculate the effective length of interaction as

$$l_\gamma \cong \frac{1}{n_\gamma \sigma_\gamma} \ .$$

Let us begin from our examples. For our first example, $mc^2\gamma$=250 *GeV*, the photon density, which corresponds to electric field strength $E \cong 1 GeV/m$, $\lambda_W \cong 1\mu m$ is $n_\gamma \cong 2.24 \cdot 10^{19} 1/cm^3$. The cross section $\sigma_C = 2.98 \cdot 10^{-25} cm^2$ yields the length

$$l_\gamma \cong \frac{1}{n_\gamma \sigma_\gamma} \cong \frac{1}{2.24 \cdot 10^{19} 2.98 \cdot 10^{-25}} \cong 1.5 \cdot 10^5 cm \ \sim 1.5\ km$$

250 *GeV* could be reached with this gradient at 250 meters, so at this distance the probability of interaction goes to 250/1500=17% or roughly 17% of all accelerated electrons will be knocked out, generating the quanta with $\hbar\omega'_{\max} \cong 170/2 \cong 85 GeV$ the end of this distance.

For $E$=10 $GeV/m$ the length of interaction shrinks to 1500/100=15 $m$. Formally for $E$=30 $GeV/m$ the length of interaction goes to 1.5 meters only. Situation is not so pessimistic however. What matters here is a strong dependence of cross section $\sigma_\gamma$ on energy. Even for such high gradients the energy after distance, say $s$=10 meter will be 300 $GeV$, $\gamma \cong 6\cdot 10^5$, $x \cong 2$ and the cross section will drop two times. See further discussion lower.

For our second example $mc^2\gamma$=50 $GeV$ (SLC) and $\sigma_C = 6.65\cdot 10^{-25} cm^2$ for the photon density, corresponding to $E \cong 30 MeV/m$, $\lambda_W \cong 10 cm$ is $n_\gamma \cong 3.2\cdot 10^{21} 1/cm^3$, and the cross section $\sigma_C = 6.65\cdot 10^{-25} cm^2$ yields the length

$$l_\gamma \cong \frac{1}{n_\gamma \sigma_\gamma} \cong \frac{1}{3.2\cdot 10^{21} 6.65\cdot 10^{-25}} \cong 470 cm = 4.7 m.$$

So after every 5 meters at the end of accelerator the ~50-$GeV$ every electron radiates the quanta with energy $\hbar\omega'_{\max} \cong 122 keV$. From the other hand the energy gain at these 5 meters will be $\Delta mc^2\gamma \equiv 30 MeV/m \times 5m \cong 150 MeV$, so the ratio is

$$\frac{\hbar\omega'_{\max}}{\Delta mc^2\gamma} \cong \frac{122 keV}{150 MeV} \cong 0.8\cdot 10^{-3}.$$

The ratio of the $\hbar\omega'/mc^2\gamma \cong 60 keV/50 GeV \cong 1.2\cdot 10^{-6}$. This remains far within the energy acceptance of the focusing structure.

For NLC the situation becomes more difficult. First of all as the field strength is planned to be about 60 $MeV/m$ and the wavelength of accelerating structure is about three times less, then, first the photon density will be about $n_\gamma \cong 3\cdot 10^{21} 1/cm^3$. For energy say $mc^2\gamma$=500$GeV$, $x$-parameter goes to $x = 4.8\cdot 10^{-6} \times 10 \times 3 \cong 1.4\cdot 10^{-4}$. Energy of the quanta will be $\hbar\omega'_{\max} \cong 500 GeV \times 1.4\cdot 10^{-4} \cong 50 MeV \times 1.4 = 72 MeV$ [10]. The length of interaction goes to

$$l_\gamma \cong \frac{1}{n_\gamma \sigma_\gamma} \cong \frac{1}{3\cdot 10^{21} 6.65\cdot 10^{-25}} \cong 500 cm.$$

The energy gain goes on this distance to $\Delta mc^2\gamma \equiv 60 MeV/m \times 5m \cong 300 MeV$, but the ratio $\frac{\hbar\omega'_{\max}}{\Delta mc^2\gamma} \cong \frac{36 MeV}{300 MeV} \cong 12\%$. The ratio of the energy losses to the *full* energy goes to $\hbar\omega'/mc^2\gamma \cong 36 MeV/500 GeV \cong 7.2\cdot 10^{-4}$ *after 5 meters of distance*[11]. This ratio goes to $1.44\cdot 10^{-2}$ after 100 meters of acceleration around 500 $GeV$. Such energy spread is beyond the edge of energy acceptance of final focus and definitely creates enormous problems at IP. So the mostly of particles will be lost for experiment.

For the same parameters of RF structure, 1.5 $TeV$ machine will have all the losses nine times the indicated above. This makes operation of NLC and all other proposed colliders

[10] Formally the process of interaction is about a multiphoton one as condition $\xi \cong 0.38 < 1$.

[11] Here we take into account that the quants radiated at $\gamma\vartheta_0 \cong 1$.

problematic beyond 250-300 $GeV$[12]. Situation is about the same despite the TESLA has lower frequency, but lower gradient and CLIC has higher frequency but higher gradient.
At last let us calculate the length of interaction for our extreme example, $mc^2\gamma$=2 $TeV$, $\gamma \cong 4 \cdot 10^6$, $\sigma_C \cong 5.6 \cdot 10^{-26} cm^2$ and the same photon density $n_\gamma \cong 2.24 \cdot 10^{19} 1/cm^3$ as in first example, $\lambda_W \cong 1\mu m$, and the length of interaction goes to

$$l_\gamma \cong \frac{1}{n_\gamma \sigma_\gamma} \cong \frac{1}{2.24 \cdot 10^{19} 5.6 \cdot 10^{-26}} \cong 8 \cdot 10^5 cm \ \sim 8 \ km.$$

In this case $\hbar\omega'_{\max} \cong mc^2\gamma \dfrac{x}{1+x} \cong 0.97 mc^2\gamma \cong 1.95 TeV$, so every act of interaction kicks out the electron, creating the gamma quanta.
So one could see that at high energy the length of interaction is growing. This may give a reason for consideration of the length dependence on more fundamental basis.

Let us express the probability of interaction as $w \cong s/l_\gamma = s \cdot n_\gamma \sigma_\gamma$. Taking into account, that energy is proportional to $s$ also, $\gamma \cong \gamma_0 + eE/mc^2 \cdot s \cong eE/mc^2 \cdot s$. In the last expression we neglected initial $\gamma_0$ value. So $x(\gamma) \cong \dfrac{4\gamma\hbar\omega_0}{mc^2} \cong \dfrac{4eE\hbar\omega_0}{(mc^2)^2} \cdot s$, and the cross section goes to

$$\sigma_\gamma \cong \frac{2\sigma_0}{x[\gamma(s)]} \{\log x[\gamma(s)] + 1/2\}$$

Taking into account expression for $n_\gamma$, $n_\gamma \cong \dfrac{1}{2\pi} \dfrac{\varepsilon_0 E^2 \lambda_W}{\hbar c}$, one can represent the length of interaction as the following

$$l_\gamma \cong \frac{1}{n_\gamma \sigma_\gamma} \cong \frac{2\pi\hbar c}{\varepsilon_0 E^2 \lambda_W} \cdot \frac{x(\gamma)}{2\sigma_0[\log x(\gamma) + 0.5]} \cong \frac{\hbar\omega_0}{\varepsilon_0 E^2} \cdot \frac{4eE\hbar\omega_0 s}{2\sigma_0 (mc^2)^2 [\log \frac{4eE\hbar\omega_0 s}{(mc^2)^2} + 0.5]}.$$

The last could be expressed as the following

$$l_\gamma \cong \left(\frac{\hbar\omega_0}{mc^2}\right)^2 \frac{e}{\varepsilon_0 \sigma_0 E} \cdot \frac{2s}{\log\left[\frac{4eE\hbar\omega_0 s}{(mc^2)^2}\right] + 0.5}.$$

One can see that the interaction length is dropping slightly faster, than linearly. Shortening of accelerated wavelength is desirable, as dependence on this is quadratic. In [1] it was shown, that even optical wavelength is possible in a laser driven microstructures. We leave further analysis of this formula lower.

## BACKGROUND ISSUES

[12] In "Zero's order Design Report" for NLC, this phenomenon not mentioned, see p.561.

At high energy the gamma-quants created will further interact with the photons in the cavity and could create an electron-positron pairs, as the limit $x$ =4.8 in overcome. This will create a shower downstream to the end and require adequate shielding.
As this interaction is going with 100% spin-flip, this effect could not affect the polarization of electron beam if it has it. There is no reduction of cross section due to polarization here.
The share $\eta(y_0, y_{\max})$ of the gammas, what have the energy in the interval from $y_{\max} = x/(1+x)$ to some reference value $y_0$ can be expressed as integral of previous formula

$$\eta(y_0, y_{\max}) = \int_{y_0}^{y_{\max}} \frac{1}{\sigma_C}\frac{d\sigma_C}{dy}\Big|_{tot} dy \cong 1.4 - \frac{0.4}{1-y_0} - 1.9\cdot y_0 + 0.7\cdot y^2 - \log(1-y_0)\,.$$

From this formula one can conclude, for example, that for $y_0 = \hbar\omega'/mc^2\gamma \cong 0.7$, or in 30% energy interval, calculated from the maximal possible, the share of all gammas is around 0.4. Or in other words, 40% of the gammas created have the energy in 30% around maximal possible. This gives an idea of background particles amount.

## THRESHOLD TO THE DAMAGE

It is interesting to compare the limit on Compton scattering effect to the level to the damage obtained by consideration atomic structure of the media [1]. We can roughly estimate the breakdown limit from the point of tunneling the electrons through the well, known in Quantum Mechanics. The tunneling probability through the barrier like shown in Fig. 2 could be expressed as the following

$$D \approx \exp\left[-\frac{4\sqrt{2m}}{3\hbar eE}\left(eU_F - e\sqrt{eE}\right)^{3/2}\right],$$

(CGSE units) where $eU_F \cong$ Fermi energy calculated from the free level and the second term defines the lowering of potential well due to Schottky effect. This probability is not sensitive to the exact view of potential well. The probability becomes of the order of unity when (SI units)

$$U_F \approx \sqrt{eE/(4\pi\varepsilon_0)}\ .$$

The last expression gives the estimation of the limiting field strength

$$E \approx \frac{4\pi\varepsilon_0 U_F^2}{e}\ .$$

Substitute here $U_F \approx 4.5V$, one can obtain $E \approx 13.7 GeV/m$. Utilization of materials with low conductance level and high outs work, $eU_F$, can increase this limit. Ideally the low conductivity materials are the best from the point of the field strength.

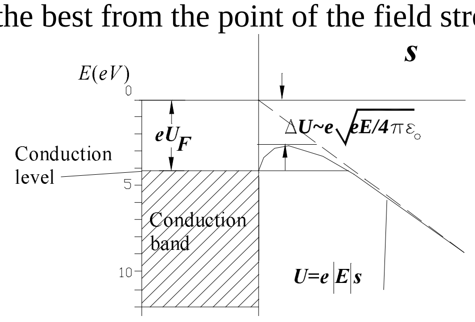

**FIGURE 2:** To the estimation of the limiting electrical field strength on the surface of metal.

Dielectric coating could help to increase the field strength. Really, in the presence of dielectric on the surface of the metal, the limiting field becomes $\varepsilon$ times, where $\varepsilon$ –is a relative dielectric permeability, the limiting electric field strength becomes higher, that for pure metal and could be estimated as

$$E \cong 4\pi\varepsilon_0 \varepsilon U_F^2 / e\,.$$

As for Diamond[13], the $\varepsilon \cong 2.1$, and one can expect the same increase of electrical field strength what gives here $E \approx 29 GeV/m$. But this is probably absolute limit. So figure ~10 *GeV/m* could be considered as a limit for a gradient in structure.

## OPTIMIZED ENERGY STRATEGY IN A LASER DRIVEN LINAC

As we are interesting in some particular laser wavelength $\lambda_0 \cong$ 1.06 $\mu m$ with energy of the quanta $\hbar\omega_0 \cong 1.17\, eV$, corresponding to Neodymium glass laser, we can express the formula in simplified way as the following

$$l_\gamma[m] \cong \frac{3.8 \cdot 10^9}{E[V/m]} \cdot \frac{2s[m]}{\log(1.8 \cdot 10^{-11} E \cdot s) + 0.5}\,.$$

This expression is valid for $x$ >1, or $\frac{4eE\hbar\omega_0 s}{(mc^2)^2} \cong 1.8 \cdot 10^{-11} E \cdot s > 1$. For $E \cong 10^9 V/m$ the distance must be $s > 55m$. Say for $s$ =100*m*, (7.1) gives $l_\gamma[m] \cong 697m$. The energy of the particles at the end of these 100 meters will be $mc^2\gamma \cong 10^{11} eV = 100 GeV$. For $s$ =700*m* gives $l_\gamma[m] \cong \frac{3.8 \cdot 1400}{\log(1.8 \cdot 7) + 0.5} \cong 1754m$. So one can see that some strategy is possible.

At initial stages of acceleration it is necessary to keep accelerating gradient low, increasing it at the end of accelerator.

The shorter wavelength of laser driven accelerator uses –the better.

## THE GAMMA FLUX IN A CIRCULAR MACHINE

The ideas described above could be applied to RF cavity in a storage ring as well. Typically the frequency of RF is about $f_0 \cong 500$ MHz, so the energy of quanta is

$$\hbar\omega_0 \cong 3.3 \cdot 10^{-25} J \cong 2.06 \cdot 10^{-6} eV\,.$$

Let us suggest that the electric field strength is about 10 *MeV/m*, what is typical for superconducting cavities. Then, the total energy stored in a cavity goes to

$$q = \int_v \frac{(\varepsilon_0 E^2 + \mu_0 H^2)}{2} dV \cong \frac{\varepsilon_0 E^2 c^3}{16 \cdot f_0^3} \cong 12J\,.$$

[13] The Diamond coating could be made from vapors.

So the number of the photons could be obtained as a ratio of previous expressions

$$N_\gamma \cong \frac{q}{\hbar\omega_0} \cong 3.6\cdot 10^{25}\,.$$

Typical volume for 500 *MHz* cavity could be taken as $V \cong 10^{-2}\,m$, so the photon density goes to

$$n_\gamma \cong \frac{N_\gamma}{V} \cong 3.6\cdot 10^{27} 1/m^3 \equiv 3.6\cdot 10^{21} 1/cm^3\,.$$

Cross section is this region is a Thomson one, $\sigma_\gamma \cong 8\pi r_0^2/3 \cong 6.65\cdot 10^{-25} cm^2$. The length of interaction respectively goes to

$$l_\gamma \cong \frac{1}{n_\gamma \sigma_\gamma} \cong 4.2m\,,$$

meanwhile the accelerating gap is something about $L_s \cong 0.3$ *m*. So in about 14 turns electron definitely kicks out the Compton quanta, i.e. probability of interaction goes to $\eta \cong L_s / l_\gamma \cong 7\%$. X –parameter equates for $mc^2\gamma \cong 5$ *GeV*, $\gamma \cong 10^4$, circulating beam to the

$$X = \frac{4\gamma^2\hbar\omega_0}{E} = \frac{4\gamma\hbar\omega_0}{mc^2} \cong 1.65\cdot 10^{-7}\,.$$

Taking into account that angle $\alpha = \pi/2$, the *x*–parameter goes to a half of previous,

$$x = X\cdot Cos^2\frac{\alpha}{2} \cong 8.2\cdot 10^{-8}\,.$$

So the energy of scattered photon goes to

$$\hbar\omega' \cong mc^2\gamma\frac{x}{1+\gamma^2\vartheta'^2+x} \cong mc^2\gamma\frac{8.2\cdot 10^{-8}}{1+\gamma^2\vartheta'^2} \cong \frac{412}{1+\gamma^2\vartheta'^2}[eV] \cong \frac{6.6\cdot 10^{-17}}{1+\gamma^2\vartheta'^2}[J]$$

Radiation of the quanta with such energy does not change the particle dynamics is accelerating ring. The losses by synchrotron radiation are about 1*MeV* in typical case, so it is not easy discover these losses by energy balance for RF source. Direct monitoring the radiation from the cavity in expecting energy spectrum could be easily done, however.

Let us calculate the gamma flux. Let the total number of electrons be $N_e$. Typically, this number is a product of (number of the trains)×(number of bunches per train) ×(number of particles in a bunch). For CESR the total number of the particles goes to $N_e \cong 10^{11}\times 9\times 4 \cong 3.6\cdot 10^{12}$. That yields the probability of Compton scattering for a single pass is about 7%. Let the revolution frequency is $f_{rev}$. Typically, say for CESR, $f_{rev} \cong 0.36MHz$. The phase $\varphi_s$ at which electron passes the cavity needs to be taken into account. This could be done by multiplication of electric field strength $E$ by the factor $Cos\varphi_s \cong 0.5$. So the number of collisions per second which create the Compton gammas is

$$\dot{n}_\gamma \cong N_\gamma\cdot 7\cdot 10^{-2}\cdot f_{rev}Cos^2\varphi_s \cong 3.6\cdot 10^{12} 7\cdot 10^{-2} 0.36\cdot 10^6\cdot 0.25 \cong 2.27\cdot 10^{16}\,photons/s\,.$$

As each photon carries the energy $\hbar\omega' \cong 3.3\cdot 10^{-17} J$, the average power caring by these photons goes to

$$P \cong \dot{n}_\gamma\hbar\omega_0 \cong 2.27\cdot 10^{16}\cdot 3.3\cdot 10^{-17} W \cong 0.7W\,.$$

Monochromaticity of scattered photons defined by energy and angular dependence of the energy. Typical energy spread in the circulating beam is something about $\Delta\gamma/\gamma \cong 5\cdot10^{-4}$ defines the width of the line. The angular spread of the radiation is going in a cone with opening angle about $\vartheta' \cong 1/\gamma \cong 10^{-4}$ with the same angular divergence around this direction. The area of this gamma-source coincides with the size of the electron/positron beam at the cavity, $<\sigma_\perp^2> \cong (\eta_{cav}\dfrac{\Delta\gamma}{\gamma})^2 + (\varepsilon\beta)^2$, where $\eta_{cav}$ –is dispersion function value at the location of the cavity, $\varepsilon$ –is the emittance of the beam and $\beta$ –is the envelope function. Typically $<\sigma_\perp^2> \cong 0.001 cm^2$, so the brightness of the source goes to $\dfrac{\dot{n}_\gamma}{<\sigma_\perp^2>/\gamma} \cong \dfrac{9.1\cdot10^{16}\cdot10^4}{10^{-3}} \cong 10^{23}\, photons/cm^2/rad$. So this radiation could be easily detected.

## CONCLUSION

The new phenomena in accelerator techniques discovered need to be taken into account when the extreme parameters of accelerating gradient desirable. The method of acceleration [1] allows reaching the extreme gradients with moderate laser power. The breakdown limit of the surface of accelerating structure estimated goes to ~10 *GeV/m*. So combination of all factors yields ~10*GeV/m* as a fundamental limit for accelerating gradient in future linear laser driven accelerators. Some relief in the condition of Compton scattering might bring optimal strategy of acceleration and utilization of optical wavelengths. At high energy of accelerated particles the cross-section of Compton scattering goes down and one can expect a possibility to accelerate up to few *TeV*.

The same process may be responsible for background effects in planning linear colliders such as NLC, JLC, CLIC, TESLA, and VLEPP, making their operation *problematic* beyond 200 *GeV*.

Scattering of electrons/positrons on the quants of RF cavity is mentioned effect in existing rings.

Acceleration of $\mu,\pi,K$ -mesons, protons/antiprotons and ions does not have this limitation, leaving ~10 *GeV/m* as a limit due to the surface damage by laser radiation, however.

The results of calculation limiting the effect from the pessimistic side, as the distribution of the losses within the angle of radiation is not taken into account. The energy of quanta is taken as half of maximal. In reality this effect will be less for low frequency accelerators.

Due to fundamental interest, direct and detailed calculations required on interaction of high-energy electrons with the photons, localized in a cavity.

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